

WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION  
FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE)

**AIMS OF THE SYLLABUS**

The aims of the syllabus are to test candidates on:

- (i) further conceptual and manipulative skills in Mathematics;
- (ii) an intermediate course of study which bridges the gap between Elementary Mathematics and Higher Mathematics;
- (iii) aspects of mathematics that can meet the needs of potential Mathematicians, Engineers, Scientists and other professionals.

**EXAMINATION FORMAT**

There will be two papers both of which must be taken.

PAPER 1: (Objective) - 1½ hours (50 marks)

PAPER 2: (Essay) - 2½ hours (100 marks)

PAPER 1 (50 marks) - This will contain forty multiple-choice questions, testing the areas common to the two alternatives of the syllabus, made up of twenty-four from Pure Mathematics, eight from Statistics and Probability and eight from Vectors and Mechanics. Candidates are expected to attempt all the questions.

PAPER 2 - This will contain two sections – A and B.

SECTION A (48 marks) - This will consist of eight compulsory questions that are elementary in type, drawn from the areas common to both alternatives as for Paper 1 with four questions drawn from Pure Mathematics, two from Statistics and Probability and two from Vectors and Mechanics.

SECTION B (52 marks) - This will consist of ten questions of greater length and difficulty consisting of three parts as follows:

PART I (PURE MATHEMATICS) - There will be four questions with two drawn from the common areas of the syllabus and one from each alternative X and Y.

PART II (STATISTICS AND PROBABILITY) - There will be three questions with two drawn from common areas of the syllabus and one from alternative X.

PART III (VECTORS AND MECHANICS) - There will be three questions with two drawn from common areas of the syllabus and one from alternative X.

Candidates will be expected to answer any four questions with at least one from each part.

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Electronic calculators of the silent, cordless and non-programmable type may be used in these papers. Only the calculator should be used; supplementary material such as instruction leaflets, notes on programming must in no circumstances be taken into the examination hall. Calculators with paper type output must not be used. No allowance will be made for the failure of a calculator in the examination.

A silent, cordless and non-programmable calculator is defined as follows:

- (a) It must not have audio or noisy keys or be operated in such a way as to disturb other candidates;
- (b) It must have its own self-contained batteries (rechargeable or dry) and not always be dependent on a mains supply;
- (c) It must not have the facility for magnetic card input or plug-in modules of programme instructions.

**DETAILED SYLLABUS**

In addition to the following topics, harder questions may be set on the General Mathematics/ Mathematics (Core) syllabus.

In the column for CONTENTS, more detailed information on the topics to be tested is given while the limits imposed on the topics are stated under NOTES.

**NOTE:** Alternative X shall be for Further Mathematics candidates since the topics therein are peculiar to Further Mathematics.

Alternative Y shall be for Mathematics (Elective) candidates since the topics therein are peculiar to Mathematics (Elective).

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AREAS COMMON TO THE TWO ALTERNATIVES			ADDITIONAL TOPICS /NOTES FOR ALTERNATIVES	
TOPIC	CONTENT	NOTES	ALTERNATIVE X (For Candidates offering Further Maths)	ALTERNATIVE Y (For Candidates offering Maths Elective)
1. Circular Measure and Radians	Lengths of Arcs of circles Perimeters of Sectors and Segments measure in radians			
2. Trigonometry	(i) Sine, Cosine and Tangent of angles (ii) Trigonometric ratios of the angles $30^{\circ}$ , $45^{\circ}$ , $60^{\circ}$ (iii) Heights and distances (iv) Angles of elevation and depression (v) Bearings, Positive and negative angles. (vi) Compound and multiple angles. (vii) Graphical solution of simple trig. equation. (viii) Solution of triangles.	For $0^{\circ} \leq \theta \leq 360^{\circ}$  Identify without use of tables.  Simple cases only.  Their use in simple Identities and solution of trig. ratios.  $a \cos x + b \sin x = c$  Include the notion of radian and trigonometric ratios of negative angles.		
3. Indices, Logarithms and Surds. (a) Indices  (b) Logarithms	(i) Elementary theory of Indices.  (ii) Elementary theory of Logarithm  $\log_a xy = \log_a x + \log_a y,$  $\log_a x^n = n \log_a x$  (iii) Applications	Meaning of $a^0, a^{-n}, a^{\frac{1}{n}}$  Calculations involving multiplication, division, power and nth roots:  $\log a^n, \log \sqrt[n]{a}, \log a^{\frac{1}{n}}$  Reduction of a relation such as $y = ax^b$ , ( $a, b$ are constants) to a linear form. $\log_{10} y = b \log_{10} x + \log_{10} a.$ Consider other examples such as $y = ab^x$ .		

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(c) Surds	<p>Surds of the form <math>\frac{a}{\sqrt{b}}</math>, <math>a\sqrt{a}</math> and <math>a + b\sqrt{n}</math></p> <p>where <math>a</math> is rational. <math>b</math> is a positive integer and <math>n</math> is not a perfect square.</p>	<p>Rationalisation of the Denominator:</p> $\frac{a + \sqrt{b}}{\sqrt{c} - \sqrt{d}}$		
(d) Sequences: Linear and Exponential sequences	<p>(i) Finite and infinite sequences</p> <p>(ii) <math>U_n = U_1 + (n - 1) d</math>, where <math>d</math> is the common difference.</p> <p>(iii) <math>S_n = \frac{n}{2} (U_1 + U_n)</math></p> <p>(iv) <math>U_n = U_1 r^{n-1}</math> where <math>r</math> is the common ratio.</p> <p>(v) <math>S_n = \frac{U_1(1 - r^n)}{1 - r}</math> ; <math>r &lt; 1</math> or <math>S_n = \frac{U_1(r^n - 1)}{r - 1}</math> ; <math>r &gt; 1</math></p>			
(e) Use of the Binomial Theorem for a positive integral index.	<p>Proof of Binomial Theorem not required.</p> <p>Expansion of <math>(a + b)^n</math></p> <p>Use of <math>(1 + x)^n \approx 1 + nx</math> for any rational <math>n</math>, where <math>x</math> is sufficiently small e.g. <math>0(0.998)^{1/3}</math></p>			
4. Algebraic Equations	<p>(a) Factors and Factorisation. Solution of Quadratic equations using:-</p> <p>(i) completing the square,</p> <p>(ii) formula.</p> <p>(a) Symmetric properties of the equation <math>ax^2 + bx + c = 0</math></p> <p>(b) Solution of two simultaneous equations where one is linear and the other quadratic.</p>	<p>The condition <math>b^2 - 4ac \geq 0</math> for the equation to have real roots.</p> <p>Sum and product of roots.</p> <p>Graphical and analytical methods permissible.</p>		

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5. Polynomials	(i) Addition, subtraction and multiplication of polynomials. (ii) Factor and remainder theorems (iii) Zeros of a polynomial function. (iv) Graphs of Polynomial functions of degree $n \leq 3$ . (v) Division of a polynomial of degree not greater than 4 by a Polynomial of lower degree.	Not exceeding degree 4		
6. Rational Functions and Partial Fractions	e.g. f: $x \rightarrow \frac{ax + b}{px^2 + qx + r}$ (i) The four basic operations. (ii) Zeros, domain and range; Sketching not required.		(iii) Resolution of rational functions into partial fractions.  Rational functions of the form $Q(x) = \frac{F(x)}{G(x)}$ $G(x) \neq 0$ where $G(x)$ and $F(x)$ are polynomials, $G(x)$ must be factorisable into linear and quadratic factors (Degree of Numerator less than that of denominator which is less than or equal to 4)	

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TOPIC	CONTENT	NOTES	ALTERNATIVE X (For Candidates offering Further Maths)	ALTERNATIVE Y (For Candidates offering Maths Elective)
7. Linear Inequalities	Graphical and Analytical Solution of simultaneous linear Inequalities in 2 variables and Quadratic inequalities.			
8. Logic	<p>(i) The truth table, using not <b>P</b> or <b>Q</b>, <b>P</b> and <b>Q</b>. <b>P</b> implies <b>Q</b>, <b>Q</b> implies <b>P</b>.</p> <p>(ii) Rule of syntax: true or false statements, rule of logic applied to arguments, implications and deductions</p>	<p>Validity of compound statements involving implications and connectives.</p> <p>Include the use of symbols: <math>\sim P</math></p> <p><math>p \vee q, p \wedge q, p \Rightarrow q</math> Use of Truth tables.</p>		
9. Co-ordinate Geometry: Straight line	<p>(a) (i) Distance between two points;</p> <p>(ii) Mid-point of a line segment;</p> <p>(iii) Gradient of a line;</p> <p>(iv) Conditions for parallel and perpendicular lines.</p> <p>(b) Equation of a line:</p> <p>(i) Intercept form;</p> <p>(ii) Gradient form;</p> <p>(iii) The general form.</p>	<p>Gradient of a line as ratio of vertical change and horizontal change.</p>		
Conic Sections	<p>(c) (i) Equation of a circle;</p> <p>(ii) Tangents and normals are required for circle.</p>	<p>(i) Equation in terms of centre and radius e.g. <math>(x-a)^2 + (y-b)^2 = r^2</math>;</p> <p>(ii) The general form: <math>x^2 + y^2 + 2gx + 2fy + c = 0</math>;</p>	(iii) Equations of parabola in rectangular Cartesian coordinates.	
10. Differentiation	(a) (i) The idea of a limit	(i) Intuitive treatment of limit. Relate to the gradient of a curve.		

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	<p>(ii) The derivative of a function.</p> <p>Application of differentiation</p> <p>(b) (i) Second derivatives and Rates of change;</p> <p>(ii) Concept of maxima and minima.</p>	<p>(ii) Its meaning and its determination from first principles in simple cases only.</p> <p>e.g. <math>ax^n + b</math>,</p> <p><math>n \leq 3, (n \in \mathbb{I})</math></p> <p>(iii) Differentiation of polynomials e.g.</p> <p><math>2x^4 - 4x^3 + 3x^2 - x + 7</math> and <math>(a + bx^n)^m</math></p> <p>(i) The equation of a tangent to a curve at a point.</p> <p>(ii) Restrict turning points to maxima and minima.</p> <p>(iii) Include curve sketching (up to cubic functions) and linear kinematics.</p>		(iv) Product and Quotient rules. Differentiation of implicit functions such as $ax^2 + by^2 = c$
<b>11. Integration</b>	<p>(i) Indefinite Integral</p> <p>(ii) Definite Integral</p>	<p>(i) Exclude <math>n = -1</math> in <math>\int x^n dx</math>.</p> <p>(ii) Integration of sum and difference of polynomials e.g. <math>4x^3 + 3x^2 - 6x + 5</math> include linear kinematics. Relate to the area under a curve.</p>		(ii) Simple problems on integration by substitution.

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	(iii) Applications of the definite integral	(iii) Plane areas and Rate of change.		(iii) Volume of solid of revolution. (iv) Approximation restricted to trapezium rule.
12. Sets	(i) Idea of a set defined by a property. Set notations and their meanings. (ii) Disjoint sets, Universal set and Complement of set. (iii) Venn diagrams, use of sets and Venn diagrams to solve problems. (iv) Commutative and Associative laws, Distributive properties over union and intersection	$\{x : x \text{ is real}\}$ , $\cup$ , $\cap$ empty set $\{ \}$ , $\emptyset$ , $\in$ , $\notin$ , $C$ , $U$ (universal set) or $A^1$ (Complement of set $A$ ).		
13. Mappings and Functions	(i) Domain and co-domain of a function. (ii) One-to-one, onto, identity and constant mapping; (iii) Inverse of a function; (iv) Composition of functions.	The notation: e.g. $f: x \rightarrow 3x + 4$ $g: x \rightarrow x^2$ where $x \in R$ .  Graphical representation of a function.  Image and the range.  Notation: $f \circ g(x) = f(g(x))$ Restrict to simple algebraic functions only.		



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<p><b>14. Matrices:</b> (a) Algebra of Matrices.</p> <p>(b) Linear Transformation</p>	<p>(i) Matrix representation</p> <p>(ii) Equal matrices</p> <p>(iii) Addition of matrices</p> <p>(iv) Multiplication of a Matrix by a scalar.</p> <p>(v) Multiplication of matrices.</p>	<p>Restrict to <math>2 \times 2</math> matrices Introduce the notation A, B, C for a matrix.</p> <p>(i) The notation <b>I</b> for the unit identity matrix.</p> <p>(ii) Zero or null matrix.</p>		<p>Some special matrices:</p> <p>(i) Reflection in the <math>x</math>-axis;</p> <p>Reflection in the <math>y</math>-axis.</p> <p>The clockwise and anti-clockwise rotation about the origin.</p> <p>(ii) Inverse of a <math>2 \times 2</math> matrix;</p> <p>(i) Restrict to the Cartesian plane;</p> <p>(ii) Composition of linear transformation;</p> <p>(iii) Inverse of a linear transformation;</p> <p>(iv) Some special linear transformations: Identity Transformation,</p> <p>Reflection in the <math>x</math>-axis</p> <p>Reflection in the <math>y</math>-axis;</p> <p>Reflection in the line <math>y = x</math></p> <p>Clockwise and anti-clockwise rotation about the origin.</p>

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(c) Determinants			<p>Evaluation of determinants of <math>2 \times 2</math> and <math>3 \times 3</math> matrices.</p> <p>Application of determinants to:</p> <p>(i) Areas of triangles and quadrilaterals.</p> <p>(ii) Solution of 3 simultaneous linear equations</p>	
15. Operations	<p>Binary Operations: Closure, Commutativity, Associativity and Distributivity, Identity elements and inverses.</p>			

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**PART II**  
**STATISTICS AND PROBABILITY**

AREAS COMMON TO THE TWO ALTERNATIVES			ADDITIONAL TOPICS /NOTES FOR ALTERNATIVES	
TOPIC	CONTENT	NOTES	ALTERNATIVE X (For Candidates offering Further Maths)	ALTERNATIVE Y (For Candidates offering Maths Elective)
1. Graphical representation of data	(i) Frequency tables. (ii) Cumulative frequency tables. (iii) Histogram (including unequal class intervals) (iv) Frequency curves and ogives for grouped data of equal and unequal class intervals.			
2. Measures of location	Central tendency; Mean, median, mode, quartiles and percentiles	Include: (i) Mode and modal group for grouped data from a histogram; (ii) Median from grouped data and from ogives; (iii) Mean for grouped data, use of an assumed mean required.		
3. Measures of Dispersion	(a) Determination of: (i) Range, Inter-Quartile range from an ogive. (ii) Variance and standard deviation.	Simple applications.  For grouped and ungrouped data using an assumed mean or true mean.		
4. Correlation	(i) Scatter diagrams  (ii) Line of fit	Meaning of correlation: positive, negative and zero correlations from scatter diagrams.  Use of line of best fit to predict one variable from another.	Rank correlation Spearman's Rank Correlation Coefficient.  Use data without ties Meaning and applications.	

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5. Probability	Meaning of probability Relative frequency Calculation of Probability. Use of simple sample spaces. Addition and multiplication of probabilities.	E.g. tossing 2 dice once, drawing balls from a box without replacement. Equally likely events and mutually exclusive events only to be used.	Probability Distribu- tion.  Binomial Probability  $P(x = r) = {}^n C_r p^r q^{n-r}$  where Probability of success = P  Probability of failure = q, $p + q = 1$ and n is the number of trials. Simple problems only.	
6. Permutations and Combinations.	Simple cases of number of arrangements on a line.          Simple cases of combination of objects.	e.g. (i) arrangement of students in a row.  (ii) drawing balls from a box. Simple problems only.  ${}^n P_r = \frac{n!}{(n-r)!}$  ${}^n C_r = \frac{n!}{r!(n-r)!}$		

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**PART III**  
**VECTORS AND MECHANICS**

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TOPIC	CONTENT	NOTES	ALTERNATIVE X (For Candidates offering Further Maths)	ALTERNATIVE Y (For Candidates offering Maths Elective)
1. Vectors	(i) Definitions of scalar and vector quantities.  (ii) Representation of Vectors.  (iii) Algebra of vectors.  (iv) Commutative, Associative and Distributive properties.  (v) The parallelogram Law.  (vi) Unit Vectors.  (vii) Position and free Vectors.	(iii) Addition and subtraction of vectors, Multiplication of vector by vectors and by scalars.  Equation of vectors.  (iv) Illustrate through diagram, diagrammatic representation. Illustrate by solving problems in elementary plane geometry e.g. concurrency of medians and diagonals.  The notation $\mathbf{i}$ for the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j}$ for the unit vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  along the $x$ and $y$ axis respectively.		

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	<p>(viii) Resolution and Composition of Vectors.</p> <p>(ix) Scalar (dot) product and its application.</p>	<p>(viii) Not more than three vectors need be composed.</p> <p>Using the dot product to establish such trigonometric formulae as</p> <p>(i) <math>\cos(a \pm b) = \cos a \cos b \pm \sin a \sin b</math></p> <p>(ii) <math>\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a</math></p> <p>(iii) <math>c^2 = a^2 + b^2 - 2ab \cos c</math></p> <p>Finding angle between two vectors.</p>		
<b>2. Statics</b>	<p>(i) Definition of a force.</p> <p>(ii) Representation of Forces.</p> <p>(iii) Composition and resolution of coplanar forces acting at a point.</p> <p>(iv) Equilibrium of particles.</p> <p>(v) Lami's theorem</p> <p>(vi) Determination of Resultant.</p>	<p>(iv) Apply to simple problems e.g. suspension of particles by strings.</p> <p>(v) Apply to simple problems on equivalent system of forces.</p>	<p>(vi) Composition and resolution of general coplanar forces on rigid bodies.</p> <p>(viii) Moments of forces.</p>	

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				Friction: Distinction between smooth and rough planes. Determination of the coefficient of friction required.
3. Dynamics	<p>(a) (i) The concepts of Motion, Time and Space.</p> <p>(ii) The definitions of displacement, velocity, acceleration and speed.</p> <p>(iii) Composition of velocities and accelerations.</p> <p>(b) Equations of motion</p> <p>(i) Rectilinear motion;</p> <p>(ii) Newton's Law of motion.</p> <p>(iii) Consequences of Newton's Laws:</p> <p>The impulse and momentum equations: Conservation of Linear Momentum.</p> <p>(iv) Motion under gravity.</p>	<p>Application of the equations of motions:</p> $V = u + at;$ $S = ut + \frac{1}{2} at^2;$ $V^2 = u^2 + 2as.$		